SINGLE VARIABLE CALCULUS EARLY TRANSCENDENTALS

SEVENTH EDITION

JAMES STEWART

McMASTER UNIVERSITY

AND

UNIVERSITY OF TORONTO

BROOKS/COLE
CENGAGE Learning™

Australia · Brazil · Japan · Korea · Mexico · Singapore · Spain · United Kingdom · United States

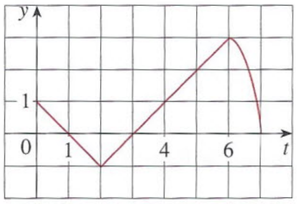
Before it was discovered, from the time of Eudoxus and Archimedes to the time of Galileo and Fermat, problems of finding areas, volumes, and lengths of curves were so difficult that only a genius could meet the challenge. But now, armed with the systematic method that Newton and Leibniz fashioned out of the Fundamental Theorem, we will see in the chapters to come that these challenging problems are accessible to all of us.

5.3 Exercises

1. Explain exactly what is meant by the statement that "differentiation and integration are inverse processes."

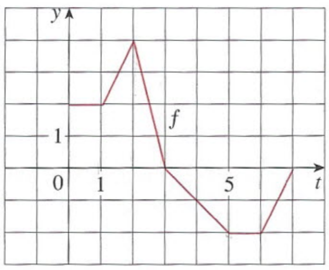
2. Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.

(a) Evaluate $g(x)$ for $x = 0, 1, 2, 3, 4, 5$, and 6.
(b) Estimate $g(7)$.
(c) Where does g have a maximum value? Where does it have a minimum value?
(d) Sketch a rough graph of g.



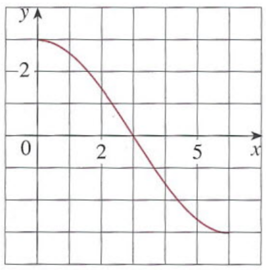
3. Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.

(a) Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$, and $g(6)$.
(b) On what interval is g increasing?
(c) Where does g have a maximum value?
(d) Sketch a rough graph of g.



4. Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.

(a) Evaluate $g(0)$ and $g(6)$.
(b) Estimate $g(x)$ for $x = 1, 2, 3, 4$, and 5.
(c) On what interval is g increasing?
(d) Where does g have a maximum value?
(e) Sketch a rough graph of g.
(f) Use the graph in part (e) to sketch the graph of $g'(x)$. Compare with the graph of f.



5–6 Sketch the area represented by $g(x)$. Then find $g'(x)$ in two ways: (a) by using Part 1 of the Fundamental Theorem and (b) by evaluating the integral using Part 2 and then differentiating.

5. $g(x) = \int_1^x t^2\,dt$

6. $g(x) = \int_0^x (2 + \sin t)\,dt$

7–18 Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

7. $g(x) = \int_1^x \frac{1}{t^3 + 1}\,dt$

8. $g(x) = \int_3^x e^{t^2 - t}\,dt$

9. $g(s) = \int_5^s (t - t^2)^8\,dt$

10. $g(r) = \int_0^r \sqrt{x^2 + 4}\,dx$

11. $F(x) = \int_x^{\pi} \sqrt{1 + \sec t}\,dt$

$$\left[\textit{Hint: } \int_x^{\pi} \sqrt{1 + \sec t}\,dt = -\int_{\pi}^{x} \sqrt{1 + \sec t}\,dt\right]$$

12. $G(x) = \int_x^1 \cos\sqrt{t}\,dt$

13. $h(x) = \int_1^{e^x} \ln t\,dt$

14. $h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4 + 1}\,dz$

15. $y = \int_0^{\tan x} \sqrt{t + \sqrt{t}}\,dt$

16. $y = \int_0^{x^4} \cos^2\theta\,d\theta$

17. $y = \int_{1-3x}^{1} \frac{u^3}{1 + u^2}\,du$

18. $y = \int_{\sin x}^{1} \sqrt{1 + t^2}\,dt$

19–44 Evaluate the integral.

19. $\int_{-1}^{2} (x^3 - 2x)\,dx$

20. $\int_{-1}^{1} x^{100}\,dx$

21. $\int_{1}^{4} (5 - 2t + 3t^2)\,dt$

22. $\int_{0}^{1} \left(1 + \frac{1}{2}u^4 - \frac{2}{5}u^9\right) du$

23. $\int_{1}^{9} \sqrt{x}\,dx$

24. $\int_{1}^{8} x^{-2/3}\,dx$

25. $\int_{\pi/6}^{\pi} \sin\theta\,d\theta$

26. $\int_{-5}^{5} e\,dx$

27. $\int_{0}^{1} (u + 2)(u - 3)\,du$

28. $\int_{0}^{4} (4 - t)\sqrt{t}\,dt$

29. $\int_{1}^{9} \frac{x - 1}{\sqrt{x}}\,dx$

30. $\int_{0}^{2} (y - 1)(2y + 1)\,dy$

31. $\int_{0}^{\pi/4} \sec^2 t\,dt$

32. $\int_{0}^{\pi/4} \sec\theta\tan\theta\,d\theta$

33. $\int_{1}^{2} (1 + 2y)^2\,dy$

34. $\int_{0}^{3} (2\sin x - e^x)\,dx$

35. $\int_{1}^{2} \frac{v^3 + 3v^6}{v^4}\,dv$

36. $\int_{1}^{18} \sqrt{\frac{3}{z}}\,dz$

37. $\int_{0}^{1} (x^e + e^x)\,dx$

38. $\int_{0}^{1} \cosh t\,dt$

39. $\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1 + x^2}\,dx$

40. $\int_{1}^{2} \frac{4 + u^2}{u^3}\,du$

41. $\int_{-1}^{1} e^{u+1}\,du$

42. $\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1 - x^2}}\,dx$

43. $\int_{0}^{\pi} f(x)\,dx$ where $f(x) = \begin{cases} \sin x & \text{if } 0 \leqslant x < \pi/2 \\ \cos x & \text{if } \pi/2 \leqslant x \leqslant \pi \end{cases}$

44. $\int_{-2}^{2} f(x)\,dx$ where $f(x) = \begin{cases} 2 & \text{if } -2 \leqslant x \leqslant 0 \\ 4 - x^2 & \text{if } 0 < x \leqslant 2 \end{cases}$

45–48 What is wrong with the equation?

45. $\int_{-2}^{1} x^{-4}\,dx = \left.\frac{x^{-3}}{-3}\right]_{-2}^{1} = -\frac{3}{8}$

46. $\int_{-1}^{2} \frac{4}{x^3}\,dx = -\left.\frac{2}{x^2}\right]_{-1}^{2} = \frac{3}{2}$

47. $\int_{\pi/3}^{\pi} \sec\theta\tan\theta\,d\theta = \sec\theta\Big]_{\pi/3}^{\pi} = -3$

48. $\int_{0}^{\pi} \sec^2 x\,dx = \tan x\Big]_{0}^{\pi} = 0$

49–52 Use a graph to give a rough estimate of the area of the region that lies beneath the given curve. Then find the exact area.

49. $y = \sqrt[3]{x},\quad 0 \leqslant x \leqslant 27$

50. $y = x^{-4},\quad 1 \leqslant x \leqslant 6$

51. $y = \sin x,\ 0 \leqslant x \leqslant \pi$

52. $y = \sec^2 x,\ 0 \leqslant x \leqslant \pi/3$

53–54 Evaluate the integral and interpret it as a difference of areas. Illustrate with a sketch.

53. $\int_{-1}^{2} x^3\,dx$

54. $\int_{\pi/6}^{2\pi} \cos x\,dx$

55–59 Find the derivative of the function.

55. $g(x) = \int_{2x}^{3x} \frac{u^2 - 1}{u^2 + 1}\,du$

$$\left[\textit{Hint: } \int_{2x}^{3x} f(u)\,du = \int_{2x}^{0} f(u)\,du + \int_{0}^{3x} f(u)\,du\right]$$

56. $g(x) = \int_{1-2x}^{1+2x} t\sin t\,dt$

57. $F(x) = \int_{x}^{x^2} e^{t^2}\,dt$

58. $F(x) = \int_{\sqrt{x}}^{2x} \arctan t\,dt$

59. $y = \int_{\cos x}^{\sin x} \ln(1 + 2v)\,dv$

60. If $f(x) = \int_0^x (1 - t^2)e^{t^2}\,dt$, on what interval is f increasing?

61. On what interval is the curve

$$y = \int_0^x \frac{t^2}{t^2 + t + 2}\,dt$$

concave downward?

62. If $f(x) = \int_0^{\sin x} \sqrt{1 + t^2}\,dt$ and $g(y) = \int_3^y f(x)\,dx$, find $g''(\pi/6)$.

63. If $f(1) = 12$, f' is continuous, and $\int_1^4 f'(x)\,dx = 17$, what is the value of $f(4)$?

64. The **error function**

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt$$

is used in probability, statistics, and engineering.

(a) Show that $\int_a^b e^{-t^2}\,dt = \frac{1}{2}\sqrt{\pi}\,[\operatorname{erf}(b) - \operatorname{erf}(a)]$.

(b) Show that the function $y = e^{x^2}\operatorname{erf}(x)$ satisfies the differential equation $y' = 2xy + 2/\sqrt{\pi}$.

65. The Fresnel function S was defined in Example 3 and graphed in Figures 7 and 8.

(a) At what values of x does this function have local maximum values?